

How to Write Like a Mathematician

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This essay introduces a basic “mathematical writing” format to help guide students who are required to write in a mathematics course. I describe the type of information that should be included in math solutions and lay out what constitutes a good written solution to a math problem. Note, a good math solution is less about the medium and more about the content (“showing your work”). This essay is NOT meant as an example of a how a POLS 309 homework solution should be formatted, although I do include a solution example to demonstrate the end how to write like a mathematician without depending on full sentences.

POLS 309 is designated as a “writing course” by Texas A&M University. When we think about writing, we often think of the essay format we’ve learned in English class:

1. Introduction and thesis statement
2. Supporting paragraph
3. Supporting paragraph
4. Conclusion

We introduce an idea, support our idea with reasoning, and then nicely wrap everything up in a concluding paragraph. This same general structure holds true when we write mathematically. When we solve math problems in a math course, we follow a similar (but different) format.

1. Write “knowns”
2. Write “unknowns”
3. Identify a strategy to get from step 1 to step 2
4. Solve it!

Overall, the first thing to notice about this strategy is that you should put a lot of effort into a math problem before you ever get an answer in step 4. Showing this effort is what your professor or TA means when they ask you to “show your work.” All “show your work” means is to communicate the thought, steps, and effort that you’ve put into your final solution (i.e. steps 1-3). Show your reasoning. Demonstrate your thinking. Do this in a way that makes sense to you. But, more importantly, show your reasoning in way that makes sense to your reader. A good trick is to pretend that your grandma will be the person reading your homework. What information do you need to include so that Granny can clearly follow along?

The key to writing mathematically when solving math problems is to clearly explain steps 1-3 so that the reader is not surprised when they see your solution in step 4. Writing clearly can look different. You could write in full sentences to clearly explain your work. Or, you could write mostly in mathematical phrases. It’s up to you. One reason it is hard to write mathematically is because there is no one right way to write mathematically. Writing mathematically is less about the medium and more about the content. The key is to “show your

work.” As long as your writing includes the information called for in steps 1-3, you will have “shown your work” and clearly explained your thinking behind your solution in step 4.

What information is each step asking for? Let’s consider each step a little more closely.

Step 1: Write “knowns”

“Knowns” are things that you know about a problem. Knowns might be data or specific variables. You might know these knowns because the problem tells you exactly what they are. However, there may also be “hidden knowns” that the problem may not explicitly state. Consider the following math problem:

Consider the following sequence: 1, 3, 7, 9. What is the standard error of the sampling distribution of the mean?

I know that my data are 1, 3, 7, and 9. But, I also *know* that $N = 4$. Hidden knowns may require you to demonstrate your knowledge of a problem. In this case, I know that N is needed to calculate the mean. Given the information provided in the problem, I know that $N = 4$ without the problem telling me that $N = 4$. I choose to write my knowns in a list, but as previously noted, you could also write in full sentences (like I have done in this paragraph):

Knowns

Data: 1, 3, 7, 9

$N = 4$

Step 2: Write “unknowns”

The hardest part of any math problem is figuring out what the problem wants you to solve for. Information you do not know, but need to know in order to solve the problem, are “unknowns.” Unknowns are often the final solution, but unknowns can also be information you need along the way in order to arrive at the final solution. Let’s return to our problem from above:

Consider the following sequence: 1, 3, 7, 9. What is the standard error of the sampling distribution of the mean?

Knowns

Data: 1, 3, 7, 9

$N = 4$

As previously mentioned, my knowns are data (1, 3, 7, 9) and $N = 4$. What are the unknowns? Right away, I know that the problem is asking me to solve for the standard error of the sampling distribution of the mean.

Unknowns

Standard error of the sampling distribution of the mean = ?

I clearly cannot solve for this unknown yet. I don't know how! Don't freak out. Lots of people panic in step 2 because they don't know how to solve for the obvious unknown in the problem. Instead of panicking, take a deep breath and ask yourself: "Are there 'hidden unknowns' I can solve for given the data provided in the problem?" Sometimes it's best to treat the knowns given to you in the problem as clues; with the data you have, what can you solve for? For example, the unknown mentions the mean. Well, I know what the mean is and know I can solve for it given my knowns (the data and N). And, if I can solve for the mean, I know I can also solve for the variance. (I also *know* that these two estimates are often used together.) I don't know how mean and variance come in to play yet, but I know that I can solve for them. Recognizing hidden unknowns can be tricky but gets easier the more problems you solve. Let's add mean and variance to our list of unknowns:

Mean = ?

Variance = ?

Step 3: Identify a strategy to get from step 1 to step 2

So, you have your knowns and your unknowns. How do you connect these two groups? Step 3 is all about creating a road map to get you from what you know to what you don't know. Given what you know, how do you solve for the unknown? What steps do you need to follow? In step 3, your goal is to construct a game plan to solve for the information you don't know, using the information you know. Consider our math problem:

Consider the following sequence: 1, 3, 7, 9. What is the standard error of the sampling distribution of the mean?

Knowns

Data: 1, 3, 7, 9

N = 4

Unknowns

Standard error of the sampling distribution of the mean = ?

Mean = ?

Variance = ?

Just like there's no one way to write mathematically, there's no one right strategy to solve a math problem. But, it is often the case that one particular strategy works better than others. Also, as students, it's usually the case that your professor has introduced a strategy in class and the homework is meant to see whether or not you can apply that specific strategy. Therefore, it's usually a good idea to think back on lectures, labs, and previous homework to ask yourself: "Have I learned any tools that are useful strategies for solving this problem?"

In the case of our POLS 309 class, we've recently learned the formula for the standard deviation of the sampling distribution:

$$\text{Standard error} = \text{standard deviation} / \text{the square root of } N$$

This is probably the best strategy because it's the direct formula for the main unknown. But, this formula also contains unknown. What is the standard deviation? If I look back at my knowns list, I do not have the standard deviation listed. Is all lost? No, not all is lost. I simply write what I know to be the formula for the standard deviation, to see if some of my knowns might help me calculate it:

$$\text{Standard deviation} = \text{the square root of the variance}$$

Aha! Variance was one of our "hidden unknowns." But do we have all of the information we need to solve for it? Let's check:

$$\text{Variance} = \text{the summation from } i = 1 \text{ to } N \text{ of each observation minus the mean squared divided by } N$$

Just like before, this formula contains an unknown: the mean. Mean was also one of the "hidden unknowns." Let's double-check what data we need to calculate the mean by writing the formula:

$$\text{Mean} = \text{the summation from } i = 1 \text{ to } N \text{ of each observation divided by } N$$

The formula for mean depends entirely on knowns. There are no more unknowns left for me to explain in this formula.

So, what now? Think strategically to make a game plan that uses all of these formulas to connect the knowns to the unknowns. We have to start with the mean, because it's the only formula that uses only knowns. Once we have the mean we can calculate the variance. Once we have the variance, we can calculate the standard deviation. Once we have the standard deviation, we can calculate the standard error of the sampling distribution.

Step 4: Solve it!

Now that you have a game plan, write out your plan, plug in numbers, and solve away.

Consider the following sequence: 1, 3, 7, 9. What is the standard error of the sampling distribution of the mean?

Knowns

Data: 1, 3, 7, 9

N = 4

Unknowns

Standard error of the sampling distribution of the mean = ?

Mean = ?

Variance = ?

a. Mean

$$\text{Mean} = (1 + 3 + 7 + 9)/4 = 5$$

b. Variance

$$\text{Variance} = ((1-5)^2 + (3-5)^2 + (7-9)^2 + (9-5)^2)/4 = 10$$

c. Standard deviation

$$\text{Standard deviation} = \text{square root of } 10 = 3.16$$

d. Standard error of the sampling distribution:

$$\text{Standard error} = 3.16 / \text{square root of } 4 = 3.16 / 2 = 1.58$$

We've solved for the unknown, but we aren't done yet. Step 4 also requires that you make clear what you think the answer to the problem is. If the data you are working with have real-life implications, include units. And, if you haven't been writing in sentences, now's the time to do it. To finish a problem, identify and (if necessary) justify your answer in a few sentences. Finally, refer back to the original question to double check you've done everything it's asked.

The standard error of the sampling distribution is 1.58.

A steam-lined example

A quick note on writing in sentences. I am clearly writing in sentences as I communicate to you about how to write like a mathematician. I have also written out mathematical formulas using words. Please don't take this activity as an endorsement that in order to write like a mathematician, you have to write in full sentences (or more directly, that every homework problem you write should follow this format.) You don't have to write in full sentences and you don't have to write your homework like I am writing this essay. I am writing in full sentences because the purpose of this document is three-fold: to describe the type of information that should be included in math solutions, to introduce a process that can be useful when solving math problems, and most importantly, to write an essay communicating how to write a solution to a math problem. I have written in full sentences because writing in full sentences best fits my purpose for writing. Similarly, you should choose to "write" in whatever style best fits your purposes. Here is a stream-lined writing example of our example problem. I recommend you follow a similar, steam-lined model when writing your homework.

Consider the following sequence: 1, 3, 7, 9. What is the standard error of the sampling distribution of the mean?

Knowns

Data: 1, 3, 7, 9

$N = 4$

Unknowns

Standard error of the sampling distribution = ?

Mean = ?

Variance = ?

$$\text{Mean} = \bar{X} = \frac{\sum_{i=1}^N X_i}{N} = \frac{1+3+7+9}{4} = 5$$

$$\text{Variance} = \sigma^2 = \frac{\sum_{i=1}^N (X_i - \bar{X})^2}{N} = \frac{(1-5)^2 + (3-5)^2 + (7-5)^2 + (9-5)^2}{4} = 10$$

$$\text{Standard deviation} = \sigma = \sqrt{\sigma^2} = \sqrt{10} = 3.16$$

$$\text{Standard error} = \frac{\sigma}{\sqrt{N}} = \frac{3.16}{\sqrt{4}} = 1.58$$

The standard deviation of the sampling mean is 1.58.

Wrapping up

There is no right way to solve a math problem but there are better and worse strategies and there are right and wrong answers. Similarly, there is no right way to write like a mathematician, but there are better and worse ways to communicate your solution. Some problems, like essay-style questions, require you to follow a format more closely aligned with the format you would use in an English or history class. Use your judgement on which format will best communicate your answer.

I've explained one potential process you can follow to solve math problems and check your writing, but this process is by no means *the way* to solve math problems and check your writing. However, the benefit of this process (identifying knowns, identifying unknowns, developing a strategy, and solving a problem) is that it captures key information that mathematical writing should communicate. Successful mathematical writing key clearly shares with your reader how you arrived at your answer.

